https://doi.org/10.7840/kics.2023.48.12.1581

Blind Equalization Based on Proximal Alternating Optimization

Wonseok Kang◆, Minsik Kim*, Daeyoung Park°

ABSTRACT

In this letter, we consider a joint channel estimation and signal detection algorithm for blind equalization in short-packet communications. We divide the non-convex blind equalization problem into two sub-problems and propose a channel estimation and signal detection algorithm based on the proximal alternating optimization. Experimental results show that the proposed algorithm provides the bit error rate (BER) performance improvement over existing methods.

Key Words : Blind equalization, proximal alternating optimization, channel estimation, detection

Ⅰ. Introduction

Blind equalization can recover transmitted signals using prior knowledge of transmitted signal statistics without training signals. Thus, it does not require pilot symbols and additional bandwidth for channel estimation. One of the most popular blind equalization algorithms is the constant modulus algorithm (CMA) for phase shift keying (PSK)[1]. However, CMA requires many transmitted symbols for algorithm convergence. To mitigate this requirement, the alternating optimization algorithm has been proposed to minimize the residual inter-symbol interference (ISI) in CMA output[2-4].

Short-packet communications become increasingly important for the next generation of ultra-reliable and low-latency communications (xURLLC). However, conventional algorithms suffer from performance degradation when the packet size is not large enough.

In this letter, we formulate a blind equalization problem to find the channel impulse response and detect the transmitted signal. Since this problem is non-convex, we divide the problem into two subproblems; recovering the transmitted symbols for given channel responses and estimating channel responses treating recovered transmitted symbols as pilot symbols. Then, we apply the proximal alternating optimization method to find the channel impulse response and the demodulated symbol jointly.

Ⅱ. System Model

We consider a multipath fading channel with L channels, where $L \geq 1$ is the oversampling ratio or the number of receive antennas. The received signal is given by

$$y_t^{(l)} = \sum_{m=0}^{K} h_m^{(l)} s_{t-m} + z_t^{(l)}, \quad l = 1, 2, \cdots, L, \qquad (1)$$

where K is the length of the channel impulse response, s_t is the element of the transmitted P symbols in M-ary quadrature amplitude modulation (QAM) for t = 0, 1, ⋯, $P+K-1$, $h^{(l)}$ is the channel impulse response of the l-th channel, and $z_t^{(l)}$ is an additive white Gaussian noise with zero mean and variance σ^2.

We estimate s_t using only the observation $y_t^{(l)}$. The blind equalization problem with joint channel estimation and symbol detection is formulated as [4]

※ This work was supported in part by Airbus Institute for Engineering Research and in part by the Institute of Information & Communications Technology Planning & Evaluation (IITP) funded by the Korea Government (MSIT) (RS-2022-00155915, Artificial Intelligence Convergence Innovation Human Resources Development (Inha University)).

◆ First Author : (ORCID:0009-0003-9949-0539) Department of Electrical and Computer Engineering, Inha University, gemstonekang94@gmail.com, 학생(석사과정), 학생회원

° Corresponding Author : (ORCID:0000-0001-8573-3526) Department of Electrical and Computer Engineering, Inha University, dpark@inha.ac.kr, 정교수, 종신회원

* (ORCID:0000-0002-0502-5464) Department of Electrical and Computer Engineering, Inha University, minsik92@gmail.com, 학생(박사과정), 학생회원

논문번호 : 202309-072-A-LU, Received September 4, 2023; Revised September 14, 2023; Accepted September 14, 2023

$$\text{(P1)} \quad \min_{s_m \in \mathbb{S}, h_m^{(l)}} \sum_{l=1}^{L} \sum_{t=0}^{P+K-1} \left| y_t^{(l)} - \sum_{m=0}^{K} h_m^{(l)} s_{t-m} \right|^2. \quad (2)$$

Since we do not know the channel order K, we consider using a linear equalization filter with N taps that are sufficiently long to equalize the channel.

Ⅲ. Proximal AO between Channel Estimation and Symbol Detection

Problem (P1) is a non-convex optimization problem, which is generally difficult to solve. Thus, we solve this problem using the proximal alternating optimization (PAO) methods[5]. We first optimize $h_m^{(l)}$ for a fixed s_t. Then, we optimize s_t for a fixed $h_m^{(l)}$. We repeat this procedure until convergence.

First, if s_t is given, then problem (P1) reduces to a conventional channel estimation problem. The least square (LS) problem with a proximal regularization is

$$\text{(P2)} \ \mathbf{H}_{n+1} = \arg\min_{\mathbf{H}} \left\| \bar{\mathbf{Y}} - \bar{\mathbf{S}}\mathbf{H} \right\|_F^2 + \eta_h \|\mathbf{H} - \mathbf{H}_n\|_F^2, \quad (3)$$

where $\bar{\mathbf{Y}} = [a_{ij}] \in \mathbb{C}^{(P+K)\times L}$ with $a_{ij} = y_{i-1}^{(j)}, \mathbf{H} = [b_{ij}] \in \mathbb{C}^{(K+1)\times L}$ with $b_{ij} = h_{i-1}^{(j)}$, and

$$\bar{\mathbf{S}} = \begin{bmatrix} s_0 & & \\ \vdots & \ddots & \\ \vdots & \ddots & s_0 \\ s_{P-1} & \ddots & \vdots \\ & \ddots & \vdots \\ & & s_{P-1} \end{bmatrix} \in \mathbb{C}^{(P+K)\times(K+1)}. \quad (4)$$

$\eta_h \geq 0$ is a proximal regularization parameter and the term $\eta_h \|\mathbf{H} - \mathbf{H}_n\|^2$ relaxes the convergence condition or enhances the convergence speed.

We assume that each transmitted symbol is randomly drawn from a constellation point set $\mathbb{S}$, which guarantees the full rank $\bar{\mathbf{S}}$ with probability 1. The closed-form solution of the estimated channel is

$$\mathbf{H}_{n+1} = (\bar{\mathbf{S}}^H \bar{\mathbf{S}} + \eta_h \mathbf{I})^{-1} (\bar{\mathbf{S}}^H \bar{\mathbf{Y}} + \eta_h \mathbf{H}_n), \quad (5)$$

which is more general than the solution given in [4]. If $\eta_h = 0$, the solution reduces to the conventional LS solution[4]. If η_h is very large, then the current channel estimate $\mathbf{H}_{n+1}$ is close to the previous channel estimate $\mathbf{H}_n$. Thus, we need to set η_h carefully to achieve the best performance.

Second, if $h_m^{(l)}$ is given, then Problem (P1) is equivalent to the MIMO detection problem. We also apply proximal regularization and the signal detection problem is

$$\text{(P3)} \ \mathbf{s}_{n+1} = \arg\min_{\{\mathbf{s} | s_t \in \mathbb{S}\}} \left\| \bar{\mathbf{y}} - \bar{\mathbf{H}}\mathbf{s} \right\|^2 + \eta_s \|\mathbf{s} - \mathbf{s}_n\|^2, \quad (6)$$

with $\bar{\mathbf{y}} = \left[\mathbf{y}^{(1)T} \ \mathbf{y}^{(2)T} \ \cdots \ \mathbf{y}^{(L)T} \right] \in \mathbb{C}^{L(P+K)\times 1}$ with $\mathbf{y}^{(l)} = [c_i] \in \mathbb{C}^{(P+K)\times 1}$ and $c_i = y_{i-1}^{(l)}$, $\mathbf{s} = [d_i] \in \mathbb{C}^{P\times 1}$ with $d_i = s_{i-1}$, and

$$\bar{\mathbf{H}} = \left[\bar{\mathbf{H}}^{(1)T} \ \bar{\mathbf{H}}^{(2)T} \ \cdots \ \bar{\mathbf{H}}^{(L)T} \right]^T \in \mathbb{C}^{L(P+K)\times P}$$

$$\bar{\mathbf{H}}^{(l)} = \begin{bmatrix} h_0^{(l)} & & & \\ \vdots & \ddots & & \\ h_K^{(l)} & \cdots & h_0^{(l)} & \\ & \ddots & \vdots & \ddots & \\ & & h_K^{(l)} & \cdots & h_0^{(l)} \\ & & & \ddots & \vdots \\ & & & & h_K^{(l)} \end{bmatrix} \in \mathbb{C}^{(P+K)\times P}. \quad (7)$$

$\eta_s \geq 0$ is a proximal regularization parameter and the term $\eta_s \|\mathbf{s} - \mathbf{s}_n\|^2$ relaxes the convergence condition or enhances the convergence speed.

Problem (P3) is NP-hard due to the finite constellation constraint $\mathbf{s} \in \mathbb{S}^P$. In general, this non-convexity can be solved by the projection operator after finding a solution that minimizes the objective function. Our proposed solution of Problem (P3) is

$$\mathbf{s}_{n+1} = \Pi_{\mathbb{S}^P} \left((\bar{\mathbf{H}}^H \bar{\mathbf{H}} + \eta_s \mathbf{I})^{-1} (\bar{\mathbf{H}}^H \bar{\mathbf{y}} + \eta_s \mathbf{s}_n) \right), \quad (8)$$

where $\Pi_{\mathbb{S}}(\mathbf{s})$ is the projection operator of each entry of $\mathbf{s}$ to the nearest constellation point in $\mathbb{S}$. If η_s = 0, the solution reduces to the solution given in [4]. If η_s is very large, then the current symbol estimate $\mathbf{s}_{i+1}$ is close to the previous symbol estimate $\mathbf{s}_i$. Thus, we need to set η_s carefully to achieve the best performance.

We solve problems (P2) and (P3) iteratively until convergence. Algorithm 1 describes the iterative channel estimation and symbol detection based on the proximal alternating optimization.

Algorithm 1. Iterative Channel Estimation and Symbol Detection Algorithm based on Proximal Alternating Optimization

1: Input: $\mathbf{y}^{(l)}, \forall l$
2: Initialize $\mathbf{s}_0$ and $\mathbf{H}_0$ with CMA
3: Repeat until convergence
4: $\mathbf{H}_{n+1} \leftarrow (\bar{\mathbf{S}}_n^H \bar{\mathbf{S}}_n + \eta_h \mathbf{I})^{-1} (\bar{\mathbf{S}}_n^H \bar{\mathbf{Y}} + \eta_h \mathbf{H}_n)$
5: $\mathbf{s}_{n+1} \leftarrow \Pi_{\mathbb{S}^P} \left((\bar{\mathbf{H}}_{n+1}^H \bar{\mathbf{H}}_{n+1} + \eta_s \mathbf{I})^{-1} (\bar{\mathbf{H}}_{n+1}^H \bar{\mathbf{y}} + \eta_s \mathbf{s}_n) \right)$
6: End

Ⅳ. Experimental Results

In this section, we evaluate the proposed blind equalization algorithm in AWGN channel with ISI. We assume that the multipath channel has a channel impulse response of length $K + 1 = 5$ and the number of receive antennas $L = 4$. We use CMA with $N = 10$ linear equalization taps for initial blind equalization.

For performance comparison, we consider the following schemes: 1) CMA [1]: One of the well-known algorithms for blind equalization, 2) AO[3,4]: The alternating optimization algorithm to minimize the residual ISI, 3) PAO: Our proposed algorithm based on proximal alternating optimization, and 4) Perfect CSI: The ideal algorithm to detect the transmitted symbols coherently with the true channel information. Throughout our evaluations, the performance metrics focus on BER to demonstrate the efficacy of each scheme under the described settings.

Figure 1 shows the BER with respect to the proximal regularization parameter η_h. Since the performance of PAO is not too sensitive to η_s, we use η_s = 0.1 as a default value. We change η_h from 0 to 200 with an interval of 10. Regardless of η_h, PAO always achieves a lower BER than AO. For the best BER performance, we chose η_h = 90 in the subsequent simulation.

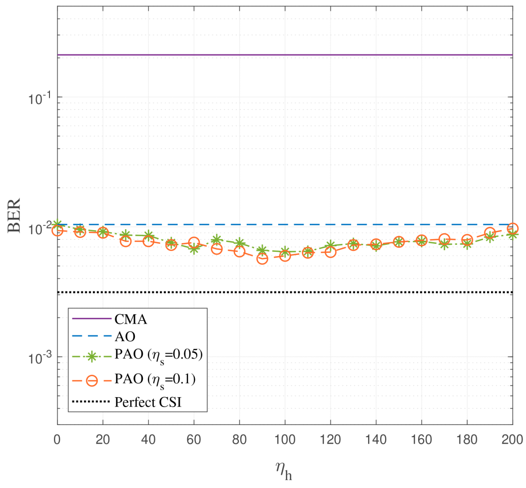

Fig. 1. BER with respect to η_h (P = 128 and SNR 4 dB).

Figure 2 shows the BER with respect to SNR. We observed the following: 1) The performance of CMA without any alternating optimization is the worst and Perfect CSI presents the BER lower bound. 2) A long packet length is required for reasonable BER performance because the BER performance improves with large P. 3) As P becomes smaller, the BER difference between AO and PAO becomes larger. This implies that the performance gain of PAO over

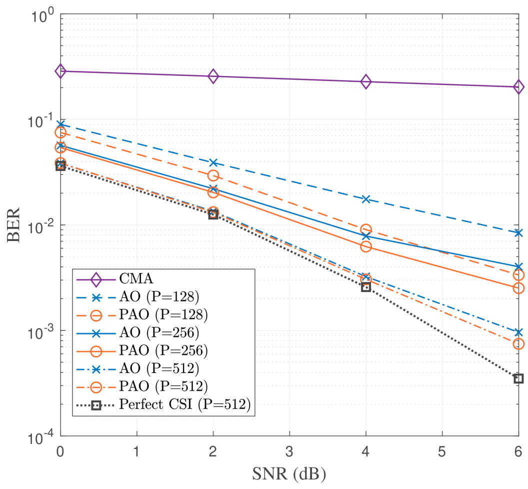

Fig. 2. BER with respect to SNR.

AO is significant in the case of short packets. 4) The performance of PAO is better than AO regardless of the *P*. In addition, the SNR gain of PAO is 2 dB when the BER is 10^{-2}.

Ⅴ. Conclusion

In this letter, we proposed a joint channel estimation and signal detection algorithm for blind equalization. We formulated a blind equalization problem to minimize the residual interference. In order to deal with the non-convexity of the blind equalization problem, we divided it into two sub-problems with proximal regularization terms. We chose the proximal regularization parameters that achieve the best BER performance. Experiments show that the proposed algorithm has a lower BER than the existing method for all considered packet sizes. In addition, the performance improvement of PAO over AO is significant when the packet length is short. Therefore, the iterative blind equalization based on PAO works well in the case of short packet communications.

References

[1] S. Haykin, *Adaptive Filter Theory*, 3rd Ed., Prentice Hall, 2014. [Online] Available: https://dl.acm.org/doi/10.5555/230061.

[2] G. Kim, J. Lee, H. Nam, and D. Park, "Iterative sparse channel estimation and demodulation for blind equalisation," *Elec. Lett.*, vol. 53, no. 21, pp. 1435-1437, 2017. [Online] Available: https://doi.org/10.1049/el.2017.2917.

[3] J. Lee, G. Kim, H. Nam, and D. Park, "Blind equalization post processing based on lock coordinate descent methods," *J. KICS*, vol. 42, no. 7, pp. 1509-1511, 2017. [Online] Available: https://doi.org/10.7840/kics.2017.42.7.1509.

[4] M. Kim, A. E. Willner, and D. Park, "Turbulence resilient free-space optical communication using iterative blind equalization," *ITC-CSCC*, 2022. [Online] Available: https://doi.org/10.1109/itc-cscc55581.2022.9894917.

[5] H. Attouch, J. Bolte, P. Redont, and A. Soubeyran, "Proximal alternating minimization and projection methods for nonconvex problems: An approach based on the kurdyka-Łojasiewicz inequality," *Math. Oper. Res.*, vol. 35, no. 2, pp. 438-457, 2010. [Online] Available: https://doi.org/10.1287/moor.1100.0449.